Landau Fermi-liquid theory

Is it possible to use perturbation theory when considering atoms and electrons in solids?

The typical distance between atoms:

$$a_0 = \frac{\hbar^2}{m_e e^2} \sim 0{,}5\,\text{Å}$$

(CGS)

– the only combination of fundamental constants ($\hbar$, m_e, e) of dimension length.

In realistic solids $r_0 \sim 1-10\,\text{Å}$ (we neglected Z)

Typical kinetic energies of electrons

$$E_k \sim \frac{e^2}{a_0} \sim 1\,\text{eV} \qquad \left(Ry = \frac{me^4}{2\hbar^2} = 13.6\,\text{eV}\right)$$

Typical interaction energies $E_{int} \sim \frac{e^2}{a_0} \sim 1\,\text{eV}$

All solid-state systems are <u>strongly interacting</u>!!!

<u>Landau Fermi-liquid theory</u>

Suggested reading: A.A. Abrikosov, "Fundamentals of the Theory of Metals", chapter 1 - beginning of 2

P. Coleman, "Many-body Physics", 7.4.2,

Also, it's possible to just google "Landau Fermi-liquid theory"

(1956)

A phenomenological theory proposed by Landau which seems to be realised by many solids

A system of strongly interacting electrons in a solid behaves like a system of weakly interacting fermions with an effective ... effective density of states, effective scattering

weakly interacting fermions with an effective mass, effective density of states, effective scattering rate different, respectively from the parameters of bare electrons
(Heat capacity , conductivity)

So, the behaviour of the electron gas is similar to the behaviour of an ideal Fermi gas

Ideal Fermi gas

Each particle state is characterised by its quasimomentum $\vec{p}$ and spin projection ± 1 ($\pm \frac{\hbar}{2}$)

Fermi distribution function

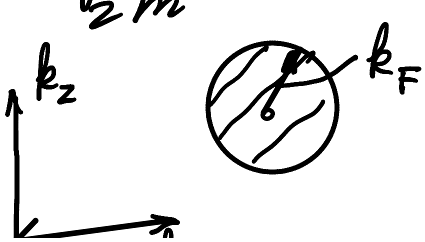
$$f = \frac{1}{e^{\frac{\varepsilon - \mu}{T}} + 1} \qquad (k_B = 1)$$

For $T \to 0$ $\quad f = 1$ for $\varepsilon < \mu$
$\qquad\qquad\quad f = 0$ for $\varepsilon > \mu$

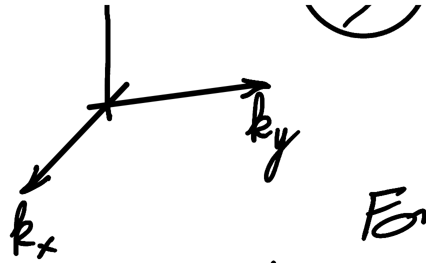
In each state $(\vec{p}, \sigma)$ there may be no more than one electron in that state (fermions), so no more than 2 electrons for each momentum

In solids $\mu \sim 1\,eV \sim 10^4\,K$, so $T \ll \mu$ usually

] $\varepsilon \sim \frac{p^2}{2m}$ (m is not the electron mass !)

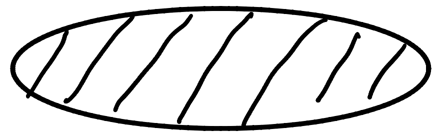


$$\hbar k_F = \sqrt{2m\mu}$$
$$(\sqrt{2m E_F})$$



$(\sqrt{2mE_F})$

For other types of dispersion



All the electrons are confined inside a surface = Fermi surface

In 3D: The total number of electrons

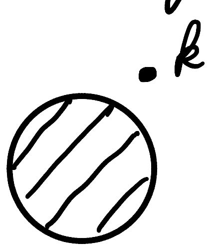
$$N = \frac{4\pi}{3} k_F^3 \cdot V \cdot 2 \frac{1}{(2\pi\hbar)^3}$$

(2 ← Spin)

$$\rightarrow k_F = \hbar \left(\frac{3\pi^2 N}{V}\right)^{\frac{1}{3}}$$

Quasiparticles

Elementary excitations

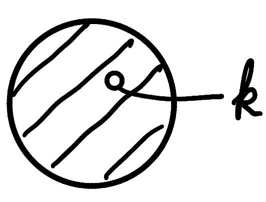


$$\xi(k) = \frac{k^2}{2m} - \frac{k_F^2}{2m} \approx$$

$$\approx v_F (k - k_F)$$

$$v_F \approx \frac{p_F}{m}$$

(moved an electron from the Fermi surface to outside of the surface)



$$\xi(k) = \frac{k_F^2}{2m} - \frac{k^2}{2m} \approx$$

$$\approx v_F (k_F - k)$$

for $|k_F - k| \ll k_F$

for $|k_F - k| \ll k_F$

$$\boxed{\xi_{\mathbf{k}} \approx v_F \, |k_F - k|}$$

// Show pictures of Fermi surfaces !!

Idea of Landau Fermi-liquid theory

$\longleftrightarrow$ quasiparticles

[Pictures of Fermi surfaces]